



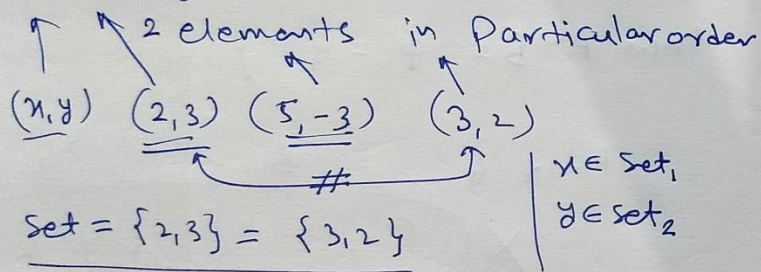
Toppers Village.

Class - 11 Chapter - 2

Relations and Functions

Exercise 2.1 - Basics

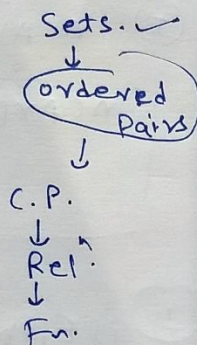
Ordered Pairs : (x, y)



Prop. of Ordered Pairs.

① $(x, y) = (a, b) \begin{cases} x = a \\ y = b \end{cases}$

ex. If $(x+3, y-1) = (7, 8)$, then
find $x \rightarrow x+3=7 \Rightarrow x=4$
 $y \rightarrow y-1=8 \Rightarrow y=9$



Cartesian Product of Sets $(A \& B) = A \times B$

$A = \text{set (nonempty)}$
 $B = \text{set (non empty)}$

$A \times B =$ Set of all possible ordered Pairs from A to B.

$$= \{(x, y) : x \in A \text{ and } y \in B\}$$

$B \times A =$ Set of all possible ordered Pairs from B to A

$$B \times A = \{(x, y) : x \in B, y \in A\}$$

Examples.

$A = \{\text{Red, yellow}\} \leftarrow \text{Colors}$

$B = \{\text{Pants, Shirt}\} \leftarrow \text{Apparels}$

$$A \times B = \{(\text{Color, App.})\}$$

$\downarrow$ this type

$$A \times B = \{(\text{Red, Pants}), (\text{Red, Shirt}), (\text{yellow, pants}), (\text{yellow, Shirt})\}$$

ex.

$$A = \{1, 2, 3\}$$

$$B = \{a, b, c\}$$

	a	b	c
1	(1,a)	(1,b)	(1,c)
2	(2,a)	(2,b)	(2,c)
3	(3,a)	(3,b)	(3,c)

↑
 $A \times B$

$$A \times B = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c), (3,a), (3,b), (3,c)\}$$

$$B \times A = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3), (c,1), (c,2), (c,3)\}$$

$$B \times A = \{(x,y) : \underline{x \in B}, y \in A\}$$

$\swarrow \quad \searrow$
 $\underline{a,b,c} \quad \underline{1,2,3}$

In above example.

$$n(A) = 3$$

$$n(B) = 3$$

$$n(A \times B) = 9 = n(B \times A)$$

Properties of ~~ordered~~ ordered Pairs.

(I) $n(A) = n_1, n(B) = n_2$ $A \times B \neq B \times A$

$$n(A \times B) = n(B \times A) = \text{no. of elements} = n_1 \cdot n_2$$

(II) If Either A or B is empty set then $A \times B = \phi = \{ \}$

$$A = \{ \}$$

$$B = \{1, 2, 3\}$$

$$\times \quad (, 1) \quad (, 2) \quad (, 3)$$

Does not make any sense.

(III) If either A or B is infinite set then $A \times B$ is also an infinite set.

$$A = \{2\}$$

$$B = \{1, 2, 3, \dots\}$$

$$A \times B = \{(2,1), (2,2), (2,3), (2,4), (2,5), \dots\}$$

(2) $\underline{1, 2, 3, \dots}$

(iv)

$$A \times A = \{ (x, y) : x \in A, y \in A \}$$

e.g.

$$\begin{aligned} A &= \{1, 2\} \longrightarrow n(A) = 2 \\ A &= \{1, 2\} \longrightarrow n(B) = 2 \end{aligned} \quad \underline{n(A \times B) = 4}$$

$$A \times A = \{ (1, 1), (1, 2), (2, 1), (2, 2) \}$$

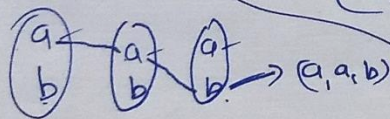
(v)

$$\begin{aligned} A \times A \times A &= \text{set of ordered triplets} \\ &= \{ (x, y, z) : x \in A, y \in A, z \in A \} \end{aligned}$$

e.g.

$$A = \{a, b\} \quad A = \{a, b\}$$

$$A \times A \times A = \{ (a, a, a), (a, a, b), (a, b, a), (a, b, b), (b, a, a), (b, a, b), (b, b, a), (b, b, b) \}$$



(IV)

$$A \times A = \{(x, y), x \in A, y \in A\}$$

e.g.

$$A = \{1, 2\} \rightarrow n(A) = 2$$

$$A = \{1, 2\} \rightarrow n(B) = 2$$

$$n(A \times B) = 4$$

$$A \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$$

(V)

$$A \times A \times A = \text{set of ordered triplets}$$

$$= \{(x, y, z) : x \in A, y \in A, z \in A\}$$

e.g.

$$A = \{a, b\} \quad A = \{a, b\}$$

$$A \times A \times A = \{(a, a, a), (a, a, b), (a, b, a), (a, b, b), (b, a, a), (b, a, b), (b, b, a), (b, b, b)\}$$

$$\begin{matrix} a & a & a \\ b & b & b \end{matrix} \rightarrow (a, a, b)$$

Toppers Village:

Exercise 2.1

$$(1) \quad \left(\frac{x}{3} + 1, y - \frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$$

$$\frac{x}{3} + 1 = \frac{5}{3} \Rightarrow \frac{x}{3} = \frac{5}{3} - 1 = \frac{2}{3} \Rightarrow x = 2$$

$$y - \frac{2}{3} = \frac{1}{3} \Rightarrow y = \frac{2}{3} + \frac{1}{3} \Rightarrow y = 1$$

(2)

$$n(A \times B) = n(A) \times n(B)$$

$$= 3 \times 3 = 9 \text{ elements}$$

(3)

$$G = \{7, 8\} \quad H = \{5, 4, 2\}$$

$$G \times H = \{(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)\}$$

(2)

$$H \times G = \{(5, 7), (4, 7), (2, 7), (5, 8), (4, 8), (2, 8)\}$$

④ ~~A~~ $P = \{m, n\}$

⑤ F $Q = \{n, m\} = \{m, n\}$

$$P \times Q = \{(m, n), (m, m), (n, n), (n, m)\}$$

⑥ T

⑦

$$\text{LHS} = A \times (B \cap \phi)$$

$$= A \times (\phi)$$

$$= \phi$$

$$= \text{RHS}$$

$\{a, b\} \cap \{x, y\}$

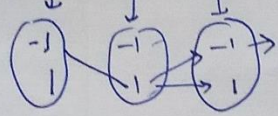
Empty

⑧

⑨ $A = \{-1, 1\}$

$$A \times A \times A = \{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1), (1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}$$

ordered triplet



⑩ $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$

~~A = ?~~ $A = ? = \{a, b\}$

$B = ? = \{x, y\}$

⑪ $A = \{1, 2\}$ $B = \{1, 2, 3, 4\}$

$C = \{5, 6\}$ $D = \{5, 6, 7, 8\}$

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$B \cap C = \phi$

$\text{LHS} = A \times (B \cap C)$

$= A \times (\phi)$

$= A \times \phi = \phi$

~~RHS =~~

$A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\}$

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

$\text{RHS} = (A \times B) \cap (A \times C)$

$= \phi = \text{LHS} \checkmark$

(ii) $A \times C$ is a subset of $B \times D$
Prove:

$$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$$

$$B \times D = \{(1, 5), (1, 6), (2, 5), (2, 6), (1, 7), (1, 8), (2, 7), (2, 8), (3, 5) \dots\}$$

clearly $A \times C$ $\subset$ $B \times D$

(8) $A = \{1, 2\}$
 $B = \{3, 4\}$

$$A \times B = \{(\underline{1, 3}), (\underline{1, 4}), (\underline{2, 3}), (\underline{2, 4})\}$$

no. of elements in $A \times B = 4 = n$

$$\text{no. of subset of } A \times B = 2^n = 2^4 = 16$$

All subsets of $A \times B \Rightarrow$

$$\begin{array}{l} \phi \\ \{(1, 3)\} \\ \{(1, 4)\} \\ \{(2, 3)\} \\ \{(2, 4)\} \end{array} \begin{array}{l} \{(1, 3), (1, 4)\} \\ \{(1, 3), (2, 3)\} \\ \{(1, 3), (2, 4)\} \\ \{(1, 4), (2, 3)\} \\ \{(1, 4), (2, 4)\} \\ \{(2, 3), (2, 4)\} \end{array}$$

$$\{(1, 3), (1, 4), (2, 4)\}$$

$$\{(1, 3), (1, 4), (2, 3)\}$$

$$\{(1, 3), (2, 3), (2, 4)\}$$

$$\{(1, 4), (2, 3), (2, 4)\}$$

$$\{(1, 3), (1, 4), (2, 3), (2, 4)\}$$

16

⑨ $n(A) = 3$

$n(B) = 2$

$(x, 1), (y, 2), (z, 1) \in A \times B$

$A = \{x, y, z\}$ ✓

$B = \{1, 2\}$ ✓

⑩ no. of elements in $A = n = 3$

$n(A \times A) = n \times n = n^2 = 9$

$\Rightarrow n^2 = 9$

$n = \pm 3$

$n = 3$

no. of elements in $A = n = 3$

$(-1, 0), (0, 1) \in A \times A$

$A = \{-1, 0, 1\}$ ✓

Basics of Exercise 2.2

Relations and Functions. — Chapter 2

Relation: ✓

A relation (R) from a non-empty set A to a non-empty set B is a subset of Cartesian Product $A \times B$.

Set Builder Form:

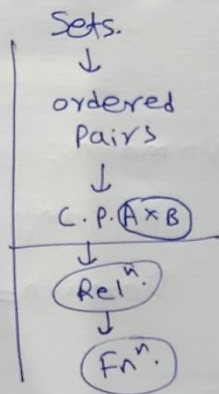
$$R = \{ (x, y) : \text{Relation (equation)} \}$$

in x & y



$$x \in A, y \in B$$

$$\begin{array}{c} R \text{ from } A \text{ to } B \\ \boxed{R: A \rightarrow B} \end{array} \quad / \quad \begin{array}{c} A \times B \\ \uparrow \end{array} \quad / \quad \begin{array}{c} x R y \\ \uparrow \end{array}$$



Here image of $x = y$

Preimage of $y = x$

Example.

Set $A = \{ \text{Amit.B., } \del{Aamir K.}, \text{Aamir K., Salman K., Akshay K.} \}$
actors.

Set $B = \{ \text{Baghban, PK, Singham, 3I} \}$

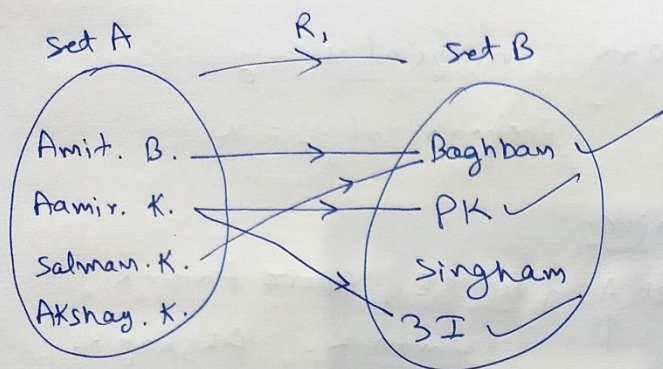
Cartesian Product $(A \rightarrow B)$ $4 \times 4 = 16$
 $A \times B = \{ \text{All ordered Pairs} \}$
 ↓
 total 16.

$R_1 = \{ (x, y) : 'x' \text{ acted in movie 'y'} \}$

$R_1: A \rightarrow B$



Set Builder



Arrow Diagram. $R_1: A \rightarrow B$

Roster Form:

$$R_1 = \left\{ (\underline{AB}, \underline{Baghban}), (\underline{Aa.K.}, \underline{PK}), (\underline{Aa.K.}, \underline{3I}), (\underline{Salman.K.}, \underline{Baghban}) \right\}$$

image of AB $\Rightarrow$ Baghban

image of Aa.K. $=$ PK, 3I

$$R_1: X \rightarrow \textcircled{Y}$$

Pre-image of Baghban = Amit. B.,
Salman. K.

Pre-image of PK = Aamir K.

Domain: set of first elements (connected) in any relation.

Domain = $\{ \underline{AB}, \underline{Aa.K.}, \underline{Salman.K.} \}$

Range: set of second elements (connected) in any relation.

~~Range~~ Range = $\{ \underline{Baghban}, \underline{PK}, \underline{3I} \}$

Codomain: the complete set $\textcircled{B}$

$$R_1: A \rightarrow \textcircled{B}$$

the second set

Codomain = $\{ \underline{Baghban}, \underline{PK}, \underline{Singham}, \underline{3I} \}$

Note: Range $\subseteq$ Codomain
 $\downarrow$ $\downarrow$
 set of only connected elements of B $\downarrow$ Complete set B

Note: R from A to B : $R: A \rightarrow B$

$$R: A \rightarrow A = \text{R from A to A} \\ = \text{R on A}$$

Example: $A = \{1, 2, 3, 4, 5, 6\}$

$$R = \{(x, y) : x = y + 2\} \text{ defined on A.}$$

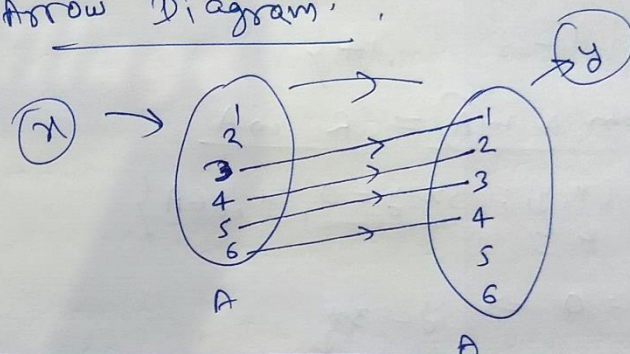
$\downarrow$ $\downarrow$
 $x \in A$ $y \in A$ $\leftarrow R: A \rightarrow A$

$x = y + 2$ $y \in A$	
$\rightarrow y = 1$ $x = 3 \in A \checkmark$	$y = 3$ $x = 5 \in A \checkmark$
$y = 2$ $x = 4 \in A \checkmark$	$y = 4$ $x = 6 \in A \checkmark$
$y = 3$	$y = 5$ $x = 7 \notin A$

$$R = \{(3, 1), (4, 2), (5, 3), (6, 4)\}$$

Roster Form:

Arrow Diagram:



$$\# \text{ No. of Relation} = \text{no. of subsets of Cartesian product } A \times B \\ = 2^{p \cdot q}$$

$$n(A) = p$$

$$n(B) = q$$

$$n(A \times B) = pq$$

E.g. $A = \{2, 3, 4\}$
 $B = \{a, b\}$

No. of relations from A to B
 $= 2^{3 \times 2} = 2^6 = 64$

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Exercise : Q.2 (All questions)

① $A = \{1, 2, 3, \dots, 14\}$

R from A to $(A) = R$ on A

$R = \{(x, y) : \underline{3x - y = 0}, x, y \in A\}$

$3x - y = 0 \Rightarrow y = 3x$
 $\downarrow \quad \downarrow$
 $y \in A \quad x \in A$

x	1	2	3	4	5
$y = 3x$	3	6	9	12	15

→ not possible.

$R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$

Domain = $\{1, 2, 3, 4\}$

Range = $\{3, 6, 9, 12\}$

Codomain = $\{1, 2, 3, \dots, 14\}$
 $\parallel$
 A

② $R : N \rightarrow (N) = R$ on N
 $\uparrow$

$R = \{(x, y) : y = x + 5\}$

$x \in N$ x is less than (4)

$x = 1, 2, 3$
 $\downarrow \quad \downarrow \quad \downarrow$
 $y = 6, 7, 8$

$R = \{(1, 6), (2, 7), (3, 8)\}$

Domain = $\{1, 2, 3\}$

Range = $\{6, 7, 8\}$

③ $A = \{1, 2, 3, 5\}$

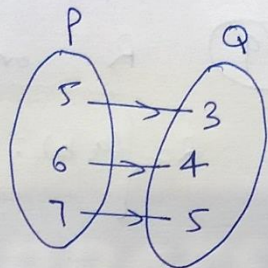
$B = \{4, 6, 9\}$

Diff. = odd = odd - Even
 $\rightarrow$ Even - odd

$R = \{(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)\}$

④

(i) Set Builder Form:



$$R = \{(x, y) : \underline{x-2=y}, x \in P, y \in Q\}$$

(ii) Roster Form:

$$R = \{(5, 3), (6, 4), (7, 5)\}$$

$$\text{Domain} = \{5, 6, 7\}$$

$$\text{Range} = \{3, 4, 5\}$$

⑤ $A = \{1, 2, 3, 4, \cancel{5}, 6\}$

$\left(\frac{b}{a}\right)$

$$R = \{(a, b) : a, b \in A, b \text{ is exactly divisible by } a\}$$

$\left(\frac{b}{a}\right)$ $\left(\frac{b}{1}\right) \xrightarrow{a \rightarrow b} \frac{2}{1} \checkmark$ $\left(\frac{6}{\cancel{A}}\right)$ $\left(\frac{2}{2}\right)$ $\left(\frac{6}{\cancel{3}}\right)$

$$R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)\}$$

$$\text{Domain} = \{1, 2, 3, 4, 6\}$$

$$\text{Range} = \{1, 2, 3, 4, 6\}$$

⑥ $R = \{(x, \underline{x+5}) : x \in \{0, 1, 2, 3, 4, 5\}\}$

$$R = \{(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)\}$$

$$\text{Domain} = \{0, 1, 2, 3, 4, 5\}$$

$$\text{Range} = \{5, 6, 7, 8, 9, 10\}$$

$$(7) R = \{ (x, x^3) : \begin{array}{l} x \text{ is prime no.} \\ \text{less than 10} \end{array} \}$$

$$x = 2, 3, 5, 7,$$

$$R = \{ (2, 2^3), (3, 3^3), (5, 5^3), (7, 7^3) \}$$

$$= \{ (2, 8), (3, 27), (5, 125), (7, 343) \}$$

$$(8) A = \{ x, y, z \} \rightarrow n(A) = 3 = p$$

$$B = \{ 1, 2 \} \rightarrow n(B) = 2 = q$$

no. of relations from A to B

$$= 2^{p \cdot q}$$

$$= 2^{3 \times 2}$$

$$= 2^6 = 64 \checkmark$$

$$(9) \quad \underline{R \text{ on } Z}$$

$$R = \{ (a, b) : a, b \in Z, \begin{array}{l} a-b \text{ is an} \\ \text{integer} \end{array} \}$$

$\downarrow$
set of integers

$$\boxed{Int_1 - Int_2 = Int.}$$

$$a \in Z \checkmark$$

$$\underline{b \in Z} \checkmark$$

$$\text{Domain} = \text{Integers} = Z$$

$$\text{Range} = Z$$

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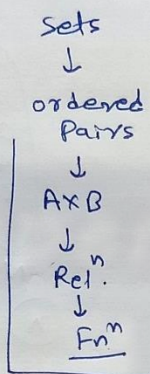
Maths

Functions | Map | Mappings:

Definition (I):

$$f: A \rightarrow B$$

A relation 'f' from a set 'A' to a set 'B' is known as function if each element of 'A' has unique (one and only one) image in 'B'.



$$f_1 = \{(1, b), (2, a), (3, c)\}$$

$$f_2 = \{(1, a), (2, a), (3, a), (4, a)\}$$

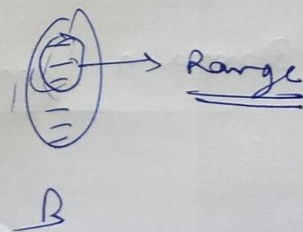
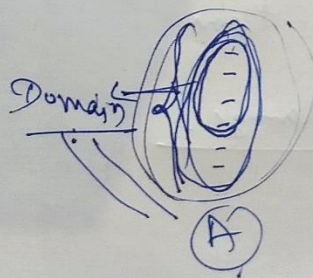
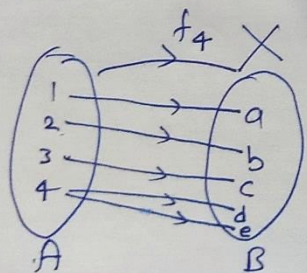
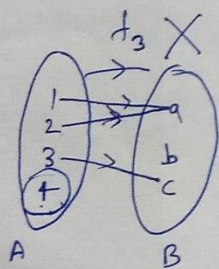
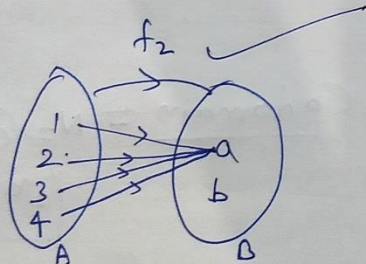
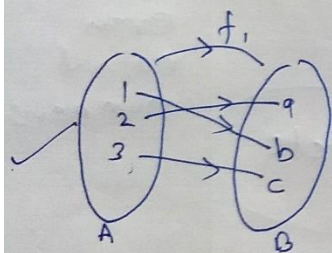
$$\times f_3 = \{(1, a), (2, a), (3, c)\} \quad A = \{1, 2, 3, \cancel{4}\} \quad B = \{a, b, c\}$$

~~(4, a)~~

$$\times f_4 = \{(1, a), (2, b), (3, c), \underline{(4, d)}, \underline{(4, e)}\}$$

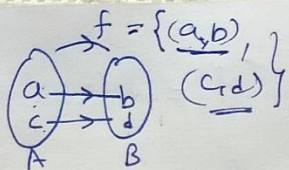
Definition - (II)

In other words, a function f from A to B such that the domain of 'f' is A and no two distinct ordered pairs in f have the same first element.



Note:

$$f: A \rightarrow B$$



(I) If 'f' is a function from A to B and $(a, b) \in f$

then

$$f(a) = b$$

~~$$P(x) = x^2 + 3$$~~

image of 'a' under 'f' = b

image of 'c' under 'f' = d

preimage of 'b' under 'f' = 'a'

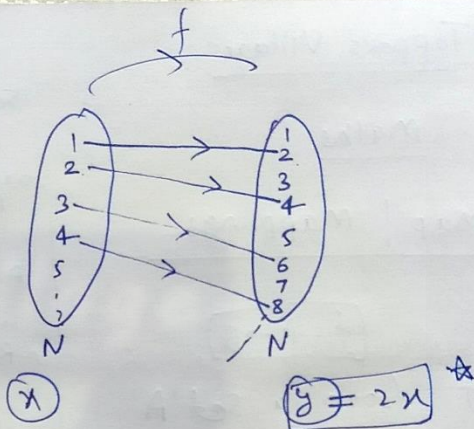
Preimage of 'd' under 'f' = c

on N

$$\text{e.g. } f = \{(x, y) : y = 2x, x, y \in N\}$$

defined from N to N then

find Domain and Range:

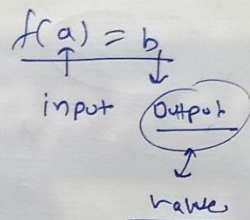
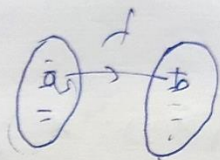


$$\text{Domain} = \{1, 2, 3, 4, 5, 6, \dots\} = N$$

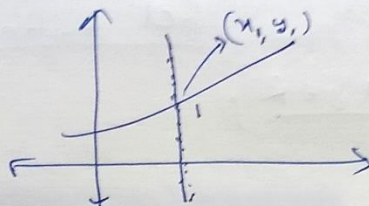
$$\text{Range} = \{2, 4, 6, 8, \dots\} = \text{Set of even natural numbers.}$$

$$\text{Codomain} = N = \text{Complete 2nd set}$$

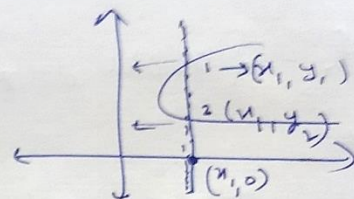
Real valued function:



those functions
whose range
is 'R' (real numbers)
or subset of 'R'.



f_1
Fun.



f_2

X
Not a Fun.

Real Function: those functions
whose domain and range
both are set of real numbers
or subset of real numbers.

Graphical Definition of Function

If vertical line does not cut
the graph of relation 'f' for
more than one point then
'f' is known as function.

Set Builder Form. interval.

$$f_1 = \{(x, y) : y = x^2, x, y \in [0, \infty)\} \rightarrow \text{yes/no}$$

$$f_2 = \{(x, y) : x = y^2, x, y \in [0, \infty)\} \rightarrow \text{yes/no}$$

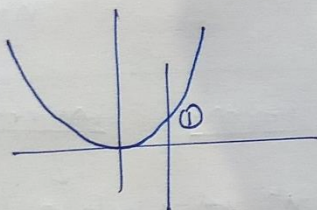
$$f_1 = \{(0, 0),$$

Set Builder

$$f_1 = \{(x, y) : y = x^2, \underline{x, y \in \mathbb{R}}\}$$

$$y = x^2$$

x	1	2	-1	-2
y	1	4	1	4



$F_{\mathbb{N}} \checkmark$

$$y = x^2$$

$\begin{pmatrix} x \\ y \end{pmatrix}$
 $\begin{matrix} \uparrow \\ -1 \end{matrix}$
 $\begin{matrix} \uparrow \\ 1 \end{matrix}$

$$\left\{ (0,0), (1,1), (2,4), (-1,1), (-2,4), \dots \right\}$$

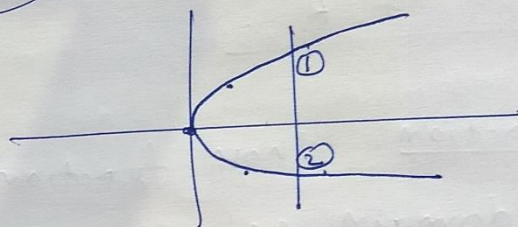


$$f_2 = \{(x, y) : x = y^2, \underline{x, y \in \mathbb{R}}\}$$

$$x = y^2$$

$$x = (-1)^2 = 1$$

x	0	1	1	4	4
y	0	1	-1	2	-2



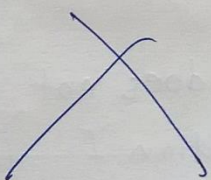
Not a $F_{\mathbb{N}}$ X

$$x = y^2$$

(x, y)

$$f_2 = \{(0,0), (\underline{1}, 1),$$

$$\underline{(1, -1)}, \dots\}$$



Toppers Village

Before Exercise 2.3

Inequality Domain Range

Most important video lecture

Inequalities ($>$, $<$, $\geq$, $\leq$)

$$\begin{array}{cccc} \frac{5 > 3}{\checkmark} & \frac{-2 < -1}{\checkmark} & \frac{0 \leq 7}{\checkmark} & \frac{x > 3}{\checkmark} \\ & & & \frac{7y^2 - 5 < 100}{\checkmark} \end{array}$$

① Let $a > b$
 $\Rightarrow -a < -b$

E.g. $7 < 10$
 $-7 > -10$

⊖ multiply
↓
Sign of inequality
change

② Let $a > b$
 $\Rightarrow \boxed{Ka > Kb}$

+K multiply
↓
Sign of inequality does not
change.

③ Reciprocal

$$\frac{x}{\uparrow} < \frac{a}{\uparrow} < \frac{b}{\uparrow}$$

Both limits required
(Same Sign)

$$\begin{array}{c|c} \begin{array}{c} \oplus \\ x < a < b \\ \Rightarrow \frac{1}{x} > \frac{1}{a} > \frac{1}{b} \end{array} & \begin{array}{c} \ominus \\ c < d < e \\ \Rightarrow \frac{1}{c} > \frac{1}{d} > \frac{1}{e} \end{array} \end{array}$$

⊖ $c < a < b$ $\oplus$
↓
Please
do
not
take
reciprocal.

$$\frac{-}{+} \quad \frac{+}{-}$$

$\frac{1}{0} = \infty = \text{Not defined}$

For e.g.

Solve. (I) $3x-1 < 2$

(II) $2 < 3-4x \leq 10$

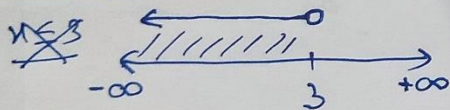
(III) $\frac{1}{2} < \frac{1}{2-3x}$

(I) $3x-1 < 2$

$\Rightarrow 3x < 3$

$\Rightarrow \frac{3x}{3} < \frac{3}{3}$

$\Rightarrow \boxed{x < 3}$



$x \in (-\infty, 3)$

Small bracket

Small bracket (Exclude)

(+3) $\downarrow$ $\oplus$ ve
No sign change
 $\leq \Rightarrow \leq$

(II) $2 < \boxed{3-4x} \leq 10$

$\Rightarrow 2-3 < -4x \leq 10-3$

$\Rightarrow -1 < \boxed{-4x} \leq 7$

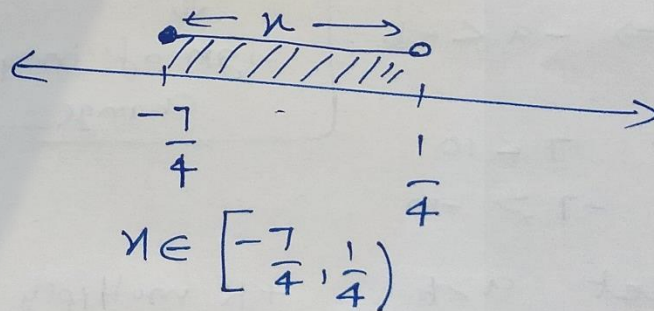
$\Rightarrow \frac{-1}{-4} > \frac{-4x}{-4} \geq \frac{7}{-4}$

(-4 $\Rightarrow$ $\ominus$ ve sign change)
Inequality str

$\Rightarrow \frac{1}{4} > x \geq -\frac{7}{4}$

$\Rightarrow -\frac{7}{4} \leq x < \frac{1}{4}$

$\frac{2 > 1}{1 < 2}$



Quadratic Inequality : (Higher Degree Inequality)

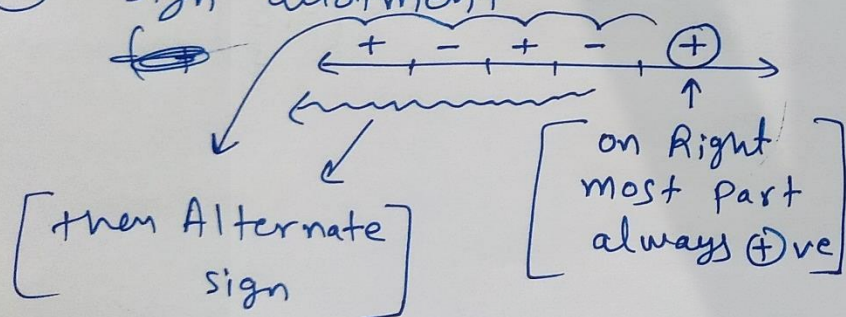
Steps. : →

(I) Make leading coefficient positive
(with other side zero)

(II) Factorize

(III) Make number line &
mark roots on it

(IV) Sign allotment



(V) Choose either +ve part
or -ve part

according to inequality

e.g. $-x^2 + 5x \leq 6$

$\Rightarrow -(-x^2 + 5x) \geq -6$ (- multiply)

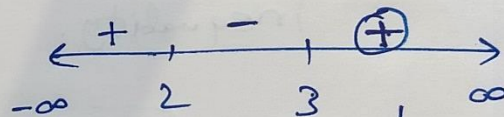
$\Rightarrow x^2 - 5x \geq -6$

$\Rightarrow x^2 - 5x + 6 \geq 0$ (I step complete)

$\Rightarrow x^2 - 2x - 3x + 6 \geq 0$

$\Rightarrow x(x-2) - 3(x-2) \geq 0$

$\Rightarrow (x-2)(x-3) \geq 0$ (II step complete)



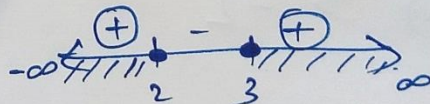
$x = 2, 3$
roots.

For e.g. 10

$(10-2)(10-3) \geq 0$
 $8 \times 7 = 56 \geq 0$

$(x-2)(x-3) \geq 0$

Positive + $\rightarrow$ choose + part.



Solution $= x \in (-\infty, 2] \cup [3, \infty)$

Next example.

Solve,

$$100 - x^2 > 0 \quad \checkmark$$

या

$$100 > x^2 \quad \checkmark$$

$$100 - x^2 > 0$$

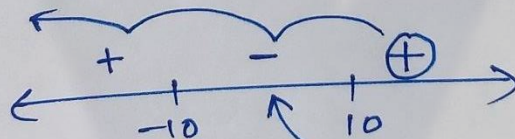
⊖ multiply. . . .

$$\Rightarrow x^2 - 100 < 0$$

$$a^2 - b^2 = (a+b) \cdot (a-b)$$

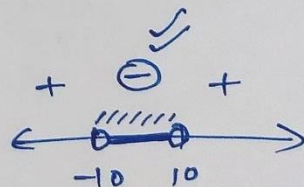
$$\Rightarrow (x+10)(x-10) < 0 \quad \star$$

$x = -10, 10$



$$(x+10)(x-10) < 0$$

⊖ ve Number



$$x \in (-10, 10)$$

Solution

Domain

Inequality $\Rightarrow$ Domain $\Rightarrow$ Range

Domain = ^{set of} all possible inputs.

Range = set of all possible outputs.

e.g. $f(x) = 3x - 7$ $f(x) = y$

↑
value of x
↑
input
↑
Domain(x)

↘
output (y)

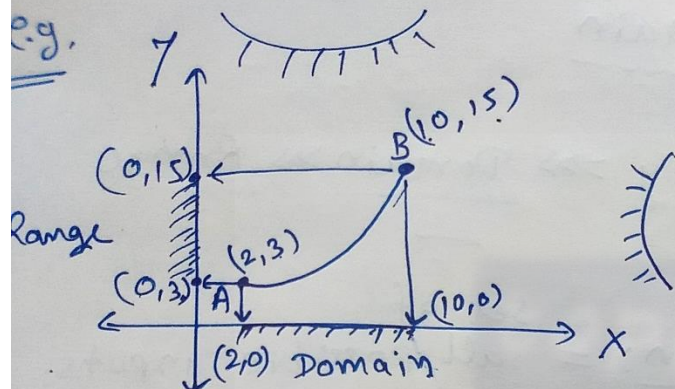
$y = f(x)$

↑
input (Domain) → X-axis

↓
Output (Range) → Y-axis

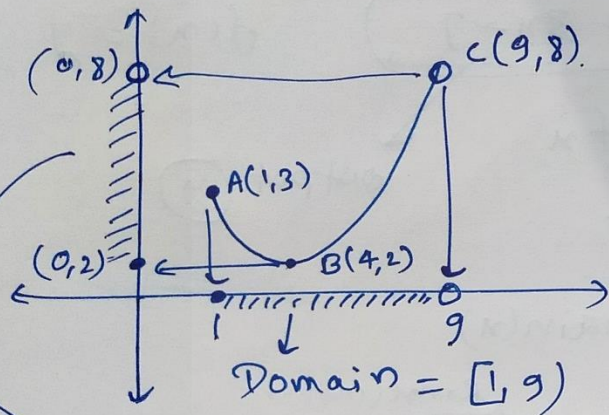
Range(output)

e.g.

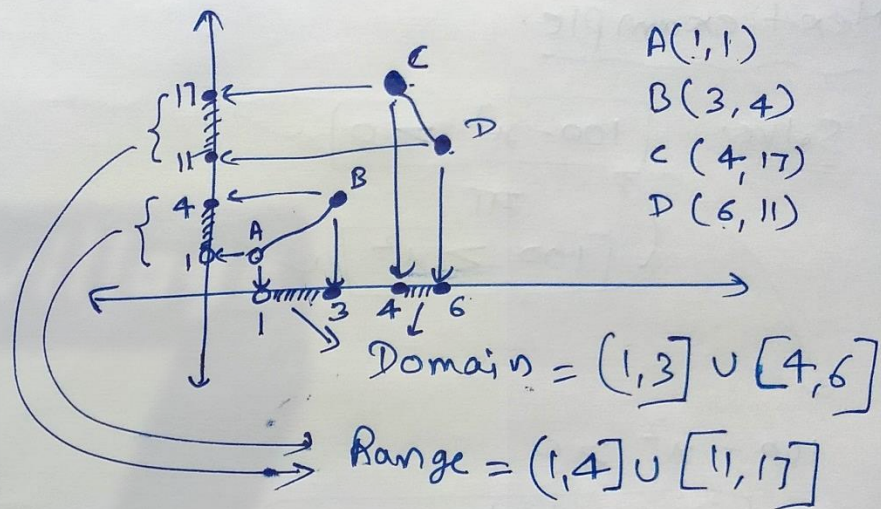


$$\text{Domain} = x \in [2, 10]$$

$$\text{Range} = y \in [3, 15]$$



$$\text{Range} = [2, 8]$$



$$A(1, 1)$$

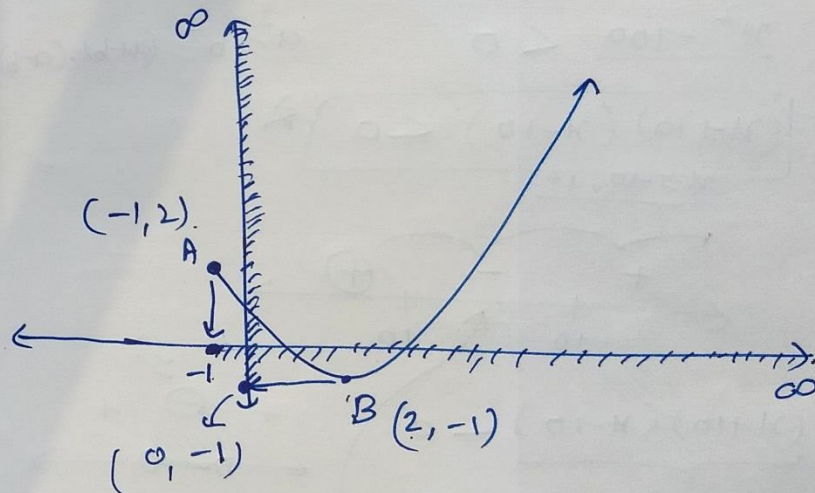
$$B(3, 4)$$

$$C(4, 17)$$

$$D(6, 11)$$

$$\text{Domain} = [1, 3] \cup [4, 6]$$

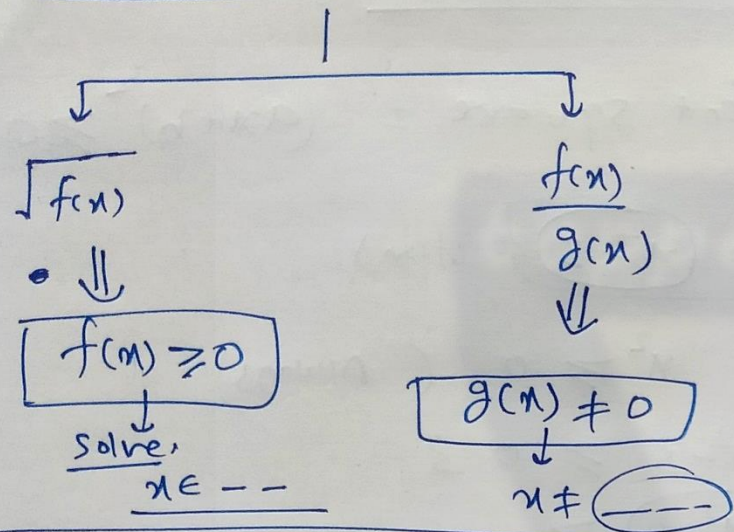
$$\text{Range} = [1, 4] \cup [11, 17]$$



$$\text{Domain} = [-1, \infty)$$

$$\text{Range} = [-1, \infty)$$

ways to find Domain : $\rightarrow$



$\frac{f(x)}{0} = \infty$
✓
Not Defined

$\times \sqrt{-} = \text{imaginary No. (Not real)}$
 $\sqrt{-1} = i = \text{iota}$
 $\sqrt{-7} = \dots$

$\sqrt{0} \checkmark$ Real

$\sqrt{+} \checkmark$ Real

for example.

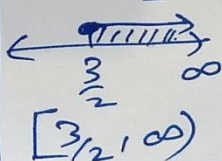
① $f(x) = \sqrt{2x-3}$

For Domain

$2x-3 \geq 0$

$\Rightarrow 2x \geq 3$

$x \geq \frac{3}{2}$

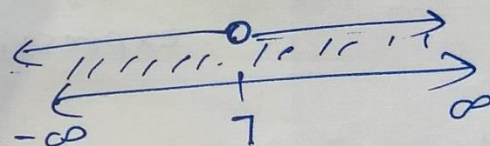


② $f(x) = \frac{2x+3}{x-7}$

for Domain $x-7 \neq 0$

$x \neq 7$

x can be anything but not 7.



$x \in (-\infty, 7) \cup (7, \infty) \checkmark$

$\mathbb{R}$

$x \in (-\infty, \infty) - \{7\} \checkmark$

$\mathbb{R}$

$x \in \mathbb{R} - \{7\} \checkmark$

Range: $\rightarrow$ output $\checkmark$
 $\downarrow$
 Value of y $\checkmark$
 $\downarrow$
 value of $f(x)$ $\checkmark$

method $\approx$ Start* with
 Known Function
 or
 Expression

$y = f(x) = x^2 - 3, \underline{x \in \mathbb{R}}$

Domain $\downarrow$ $\&$ Range.
 $x \in \mathbb{R}$

Domain = $\mathbb{R}$ $\checkmark$

Range $y = f(x) = x^2 - 3$
 $x \in \mathbb{R}$

Known $= x^2 =$ equal to zero
 or positive.

But $x^2 \neq$ negative.

Perfect square = $(ax+b)^2 \geq 0$

$y = (x^2 - 3) = f(x)$

$x^2 \geq 0$ (Always)

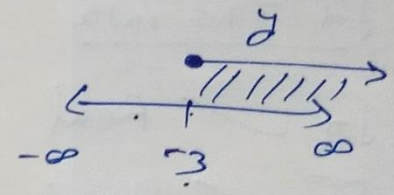
$\Rightarrow x^2 - 3 \geq 0 - 3$

$\Rightarrow (x^2 - 3) \geq -3$

$f(x) \geq -3$

या

$y \geq -3$



~~$y \in \mathbb{R}$~~ $y \in [3, \infty)$

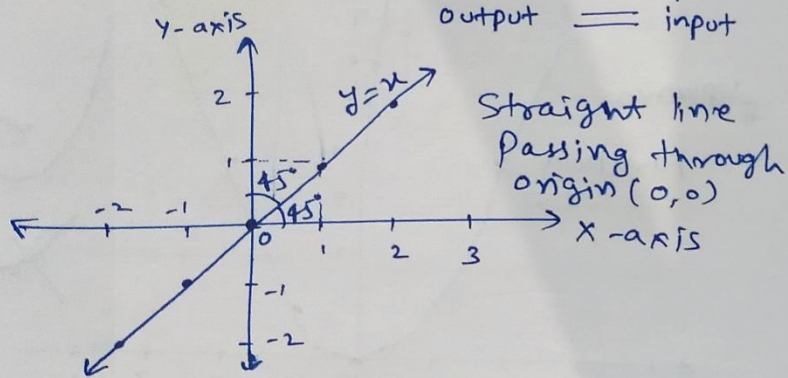
Range = $[-3, \infty)$

Some Functions and their Graphs

Name	Expression	Graph	★ Domain Range
------	------------	-------	----------------------

(i) Identity Function:

$f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $y = f(x) = x$
 $\downarrow \qquad \qquad \qquad \uparrow$
 output = input



x	0	1	2	-1	-2
y	0	1	2	-1	-2

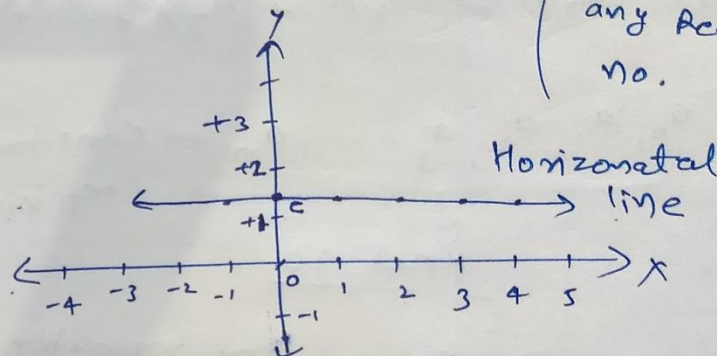
Domain = $\mathbb{R}$
 Range = $\mathbb{R}$

Codomain = $\mathbb{R}$

(ii) Constant Function:

$f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = y = c$
 $\uparrow \qquad \qquad \qquad \downarrow$

(Constant any Real no.)



x	0	1	2	3	4	-1
y	c	c	c	c	c	c

constant output

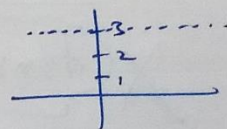
Domain = $\mathbb{R}$

Range = $\{c\}$ = Singleton set.

e.g. $y = f(x) = 3$

Domain = $\mathbb{R}$

Range = $\{3\}$



(iii) Polynomial Functions

$f: \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_nx^n$$

$a_0, a_1, a_2, a_3, \dots, a_n \rightarrow$ Constant Coefficients.
Real no.

$n =$ power of the variable ' x '
 $=$ whole no. $= \{0, 1, 2, 3, \dots\}$

e.g. $f(x) = 2x + 3$

$f(x) = x$

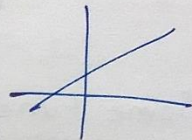
$f(x) = 3x^3 - 5x^2 + 7x - 8$

~~$f(x) = x^{2/3} - x^{100} + x^2 + 1$~~
 $2/3 \notin \mathbb{W}$

Note: Linear Functions

$y = f(x) = \frac{ax+b}{a \neq 0}$

Straight line



e.g.

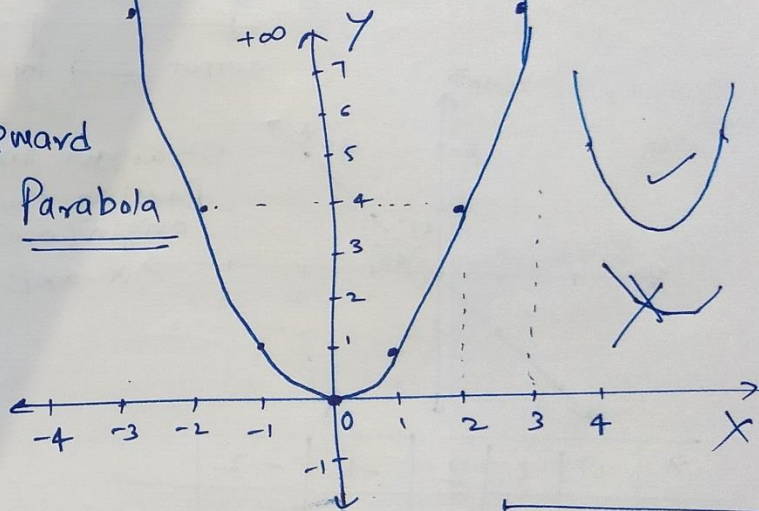
① $y = x^2$

$f: \mathbb{R} \rightarrow \mathbb{R}$

x	0	1	2	3	4	-1	-2	-3	-4	...
y	0	1	4	9	16	1	4	9	16	...

Upward

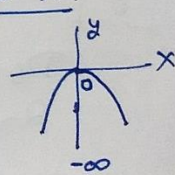
Parabola



Domain $= \mathbb{R}$

Range $= [0, \infty)$

$y = -x^2$



e.g. ② $y = f(x) = x^3$

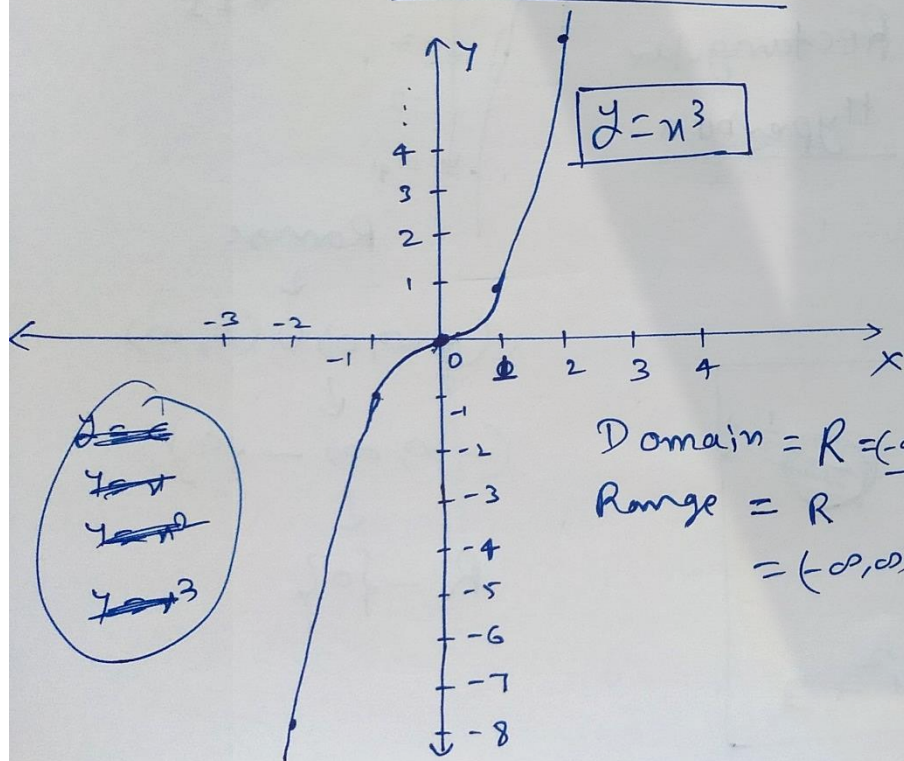
$$f: \mathbb{R} \rightarrow \mathbb{R}$$

↑
input

$$y = x^3$$

x	0	1	2	3	-1	-2	-3	...
y	0	1	8	27	-1	-8	-27	

$$(-1)^3 = -1 \cdot -1 \cdot -1 = -1$$



Domain = $\mathbb{R} = (-\infty, \infty)$
 Range = $\mathbb{R} = (-\infty, \infty)$

(iv) Rational Functions: $= \frac{f(x)}{g(x)}$, $g(x) \neq 0$

$f(x), g(x) \rightarrow$ Polynomial F_n .

e.g. $f(x) = \frac{2x+3}{3-x}$

Domain $\ni x \in \mathbb{R} - \{3\}$

$3-x \neq 0$

$x \neq 3$

$\frac{x}{0} = \infty$

e.g. $y = \frac{1}{x} = f(x)$

$x \neq 0$

Domain = $\mathbb{R} - \{0\}$

$f: \mathbb{R} - \{0\} \rightarrow \mathbb{R} - \{0\}$

$$y = \frac{1}{x} = f(x)$$

$$f: \mathbb{R} - \{0\} \rightarrow \mathbb{R} - \{0\}$$

x	0.25	0.5	1	2	-0.25	0.5	-1	-2
y	4	2	1	0.5	-4	-2	-1	-0.5

$$y = \frac{1}{x} = \frac{1}{0.25} = \frac{100}{25} = 4$$

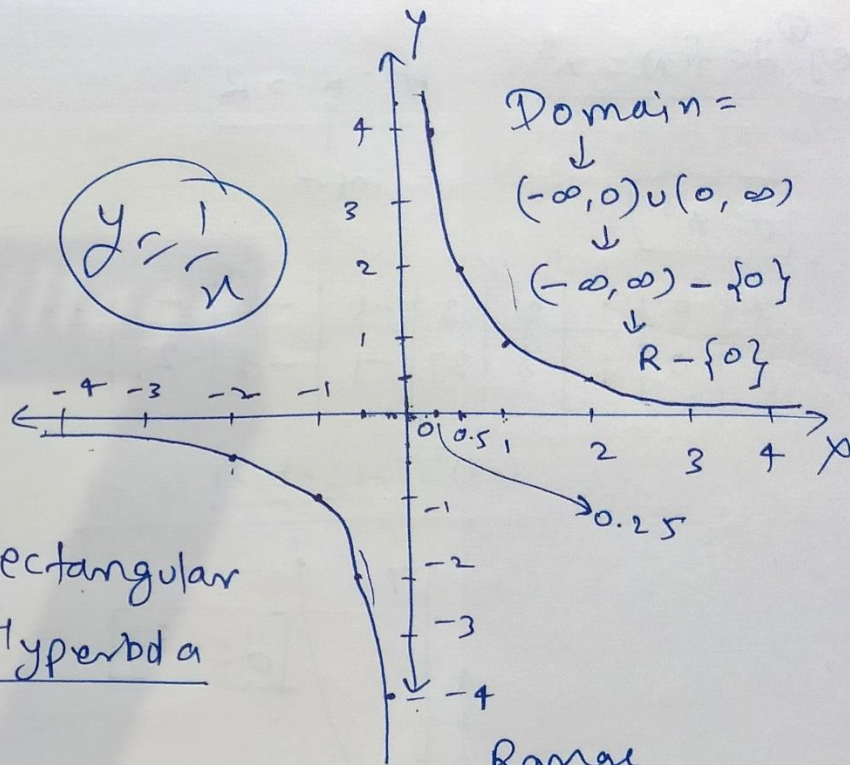
$$y = \frac{1}{x} = \frac{1}{0.5} = \frac{10}{5} = 2$$

$$y = \frac{1}{1} = 1$$

$$y = \frac{1}{2} = 0.5$$

$$y = \frac{1}{x}$$

Rectangular Hyperbola



Domain =

$$(-\infty, 0) \cup (0, \infty)$$

$$(-\infty, \infty) - \{0\}$$

$$\mathbb{R} - \{0\}$$

Range

$$(-\infty, 0) \cup (0, \infty)$$

$$(-\infty, \infty) - \{0\}$$

$$\mathbb{R} - \{0\}$$

Note: Irrational F_n^m .

$$f(x) = \sqrt{x} \rightarrow x \geq 0$$

$$f(x) = x^{2/3}$$

$$f(x) = \sqrt{(7x-3) + x^2 + 3}$$

$$\frac{1}{2} \cdot \frac{1}{3}$$

Some functions and their graphs:

Name	Expression	Graph	Domain Range
------	------------	-------	-----------------

$$y = f(x) = |x| = \begin{cases} x & , x \geq 0 \\ -x & , x < 0 \end{cases}$$

positive
& zero

Negative

$$+ (\text{Positive}) = \oplus$$

$$- (\text{Negative}) = \oplus$$

⑤ The Modulus Function (Absolute Value Functions)

$f: \mathbb{R} \rightarrow \mathbb{R}$ ~~define~~ defined
by $f(x) = |x|$ ✓

e.g. $f(2) = |2| = 2$

$$f(-2) = |-2| = 2 = -(-2)$$

$$f(0) = |0| = 0$$

$$f(-3.8) = |-3.8| = 3.8$$

$$x = -4$$

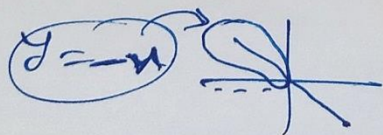
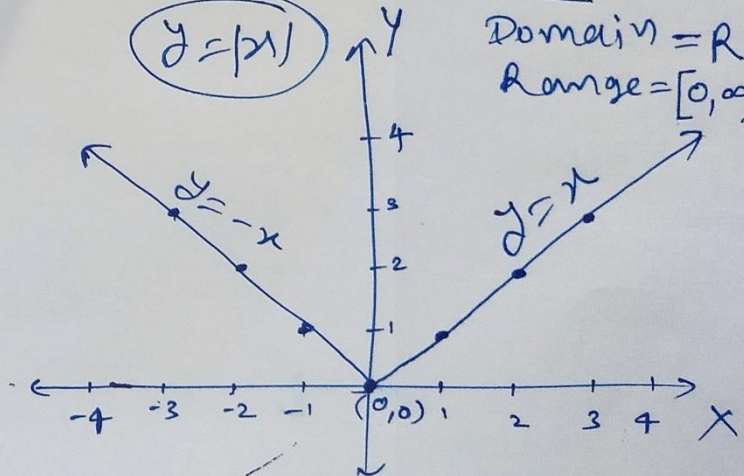
$$f(-4) = |-4| = \begin{cases} -(-4) & , -4 < 0 \end{cases} \\ = 4 = 4$$

$$y = |x|$$

x	y
0	0
1	1
2	2
-1	1
-2	2
-3	3
3	3

$$y = |x|$$

Domain = $\mathbb{R}$
Range = $[0, \infty)$



Toppers Village

(VI) Signum Function:

$f: \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$y = f(x) = \begin{cases} +1 & \text{if } x > 0 \text{ (positive)} \\ 0 & \text{if } x = 0 \text{ (zero)} \\ -1 & \text{if } x < 0 \text{ (Negative)} \end{cases}$$

$$\text{sgn}(x) = \begin{cases} +1 & \text{if } x > 0 \text{ (positive)} \\ 0 & \text{if } x = 0 \text{ (zero)} \\ -1 & \text{if } x < 0 \text{ (Negative)} \end{cases}$$

e.g. $f(2) = +1 = \text{sgn}(2)$ ($\because 2 > 0$)
 $f(-3) = -1 = \text{sgn}(-3)$ ($\because -3 < 0$)

$$\text{sgn}(100) = +1$$

$$\text{sgn}(-20) = -1$$

$$\text{sgn}(\pi) = +1$$

$$\text{sgn}(0) = 0$$

$$\text{sgn}(-\pi) = -1$$

$$x > 0$$

$$x = 3$$

$$\frac{|3|}{3} = \frac{3}{3} = 1$$

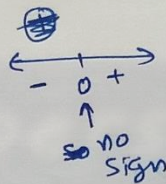
$$x < 0$$

$$x = -10$$

$$\frac{|-10|}{-10} = \frac{10}{-10} = -1$$

$$\frac{|x|}{x} = \frac{x}{|x|}$$

$$-2 = \frac{-2}{1} = -2$$

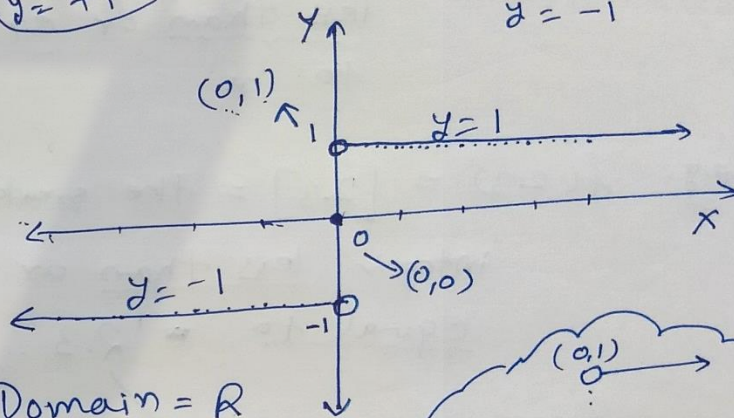


Graph of $\text{sgn}(x)$:

x	0	1	2	3	4	...	-1	-2	-3	-4	...
$\text{sgn}(x) = y$	0	1	1	1	1		-1	-1	-1	-1	

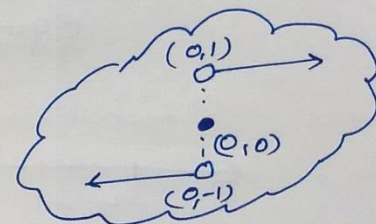
$x = 0.001$
 $y = +1$

$x = -0.0001$
 $y = -1$



Domain = $\mathbb{R}$

Range = $\{1, 0, -1\}$



$$y = \text{sgn}(x) = f(x) = \begin{cases} 1 & , x > 0 \\ 0 & , x = 0 \\ -1 & , x < 0 \end{cases}$$

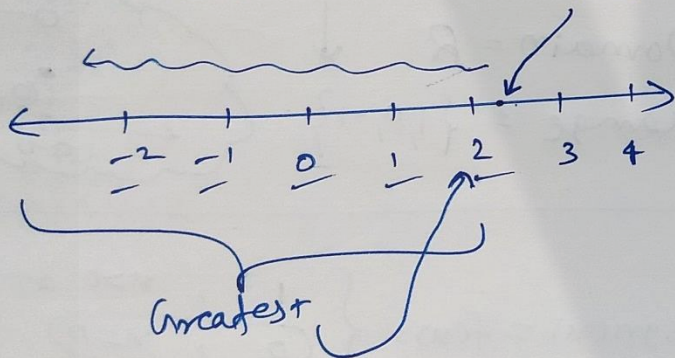
$$= \begin{cases} 0 & , x = 0 \\ \frac{|x|}{x} & , x \neq 0 \end{cases}$$

VII Greatest integer Function (GIF)

$f: \mathbb{R} \rightarrow \mathbb{R}$ defined by

$y = f(x) = [x] =$ the greatest integer less than or equal to ' x '

e.g. $f(2.3) = [2.3] =$ the greatest integer less than or equal to ' 2.3 '.

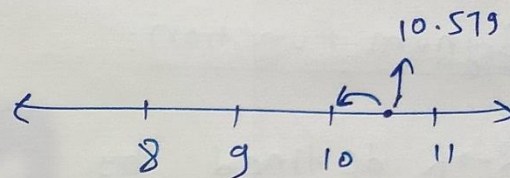


$$f(2.3) = [2.3] = 2 \quad \checkmark$$

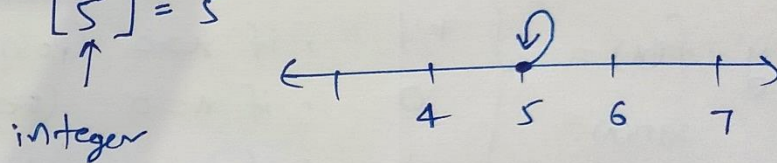
$$[2.4] = 2$$

$$[2.9] = 2$$

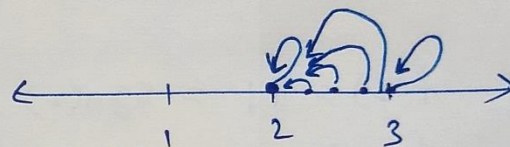
e.g. $[10.579] = 10$



e.g. $[5] = 5$

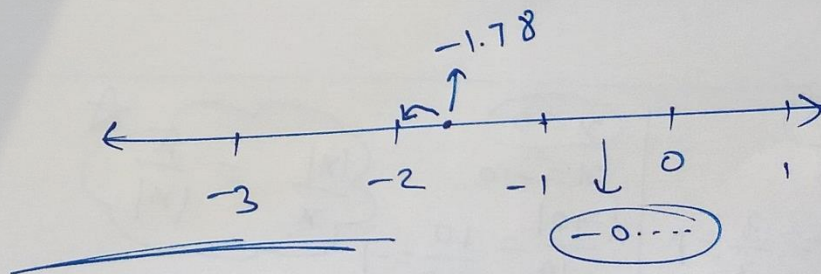


e.g.



$$[2.99999] = 2$$

e.g. $[-1.78] = -2$



$$\begin{aligned} [0] &= 0 \\ [0.1] &= 0 \\ [0.2] &= 0 \\ &\vdots \\ [0.9] &= 0 \end{aligned}$$

$$[0.9] = 0$$

$$[1] = 1$$

$$[1.1] = 1$$

$$[1.2] = 1$$

$$[1.999] = 1$$

$$[2] = 2$$

$$[2.1] = 2$$

$$[2.2] = 2$$

$$[2.99] = 2$$

$$[3] = 3$$

$$[3.001] = 3$$

$$[3.99] = 3$$

$$[4] = 4$$

$$[4.01] = 4$$

$$[-0.001] = -1$$

$$[-0.2] = -1$$

$$[-0.9] = -1$$

$$[-1] = -1$$

$$[-1.1] = -2$$

$$[-1.5] = -2$$

$$[-1.9] = -2$$

$$[-2] = -2$$

$$[-2.001] = -3$$

$$[-2.2] = -3$$

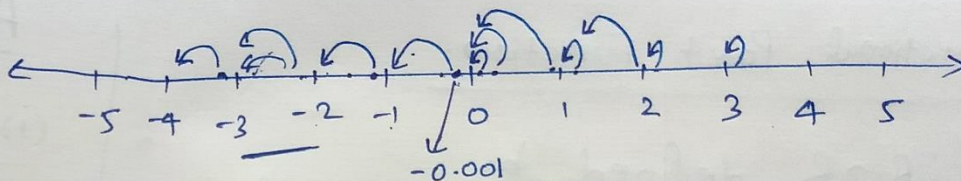
$$[-2.3] = -3$$

$$[-2.99] = -3$$

$$[-3] = -3$$

$$[-3.1] = -4$$

$$[\pi] = 3$$



$$[I] = I$$

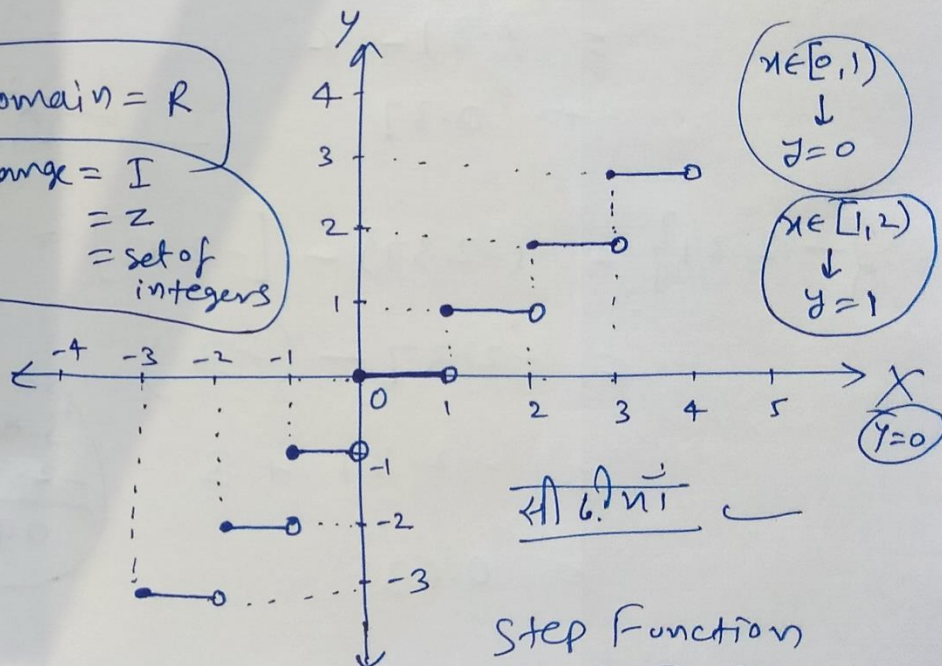
↑
integer

$$y = f(x) = \begin{cases} -3, & x \in [-3, -2) \\ -2, & x \in [-2, -1) \\ -1, & x \in [-1, 0) \\ 0, & x \in [0, 1) \rightarrow 0 \leq x < 1 \\ 1, & x \in [1, 2) \rightarrow 1 \leq x < 2 \\ 2, & x \in [2, 3) \rightarrow 2 \leq x < 3 \\ 3, & x \in [3, 4) \end{cases}$$

$$= [x] =$$

$$\text{Domain} = \mathbb{R}$$

$$\begin{aligned} \text{Range} &= \mathbb{I} \\ &= \mathbb{Z} \\ &= \text{set of integers} \end{aligned}$$



$$[x] = \lfloor x \rfloor$$

Step Function

Fractional Part Function.

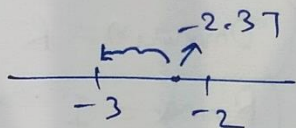
$f: \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$y = f(x) = \boxed{\{x\} = x - [x]} \quad \star$$

$\rightarrow \text{WIF}$

$$\begin{aligned}\{2.37\} &= 2.37 - [2.37] \rightarrow \text{WIF} \\ &= 2.37 - 2 \\ &= 0.37\end{aligned}$$

$$\begin{aligned}\{-2.37\} &= (-2.37) - [-2.37] \\ &= -2.37 - (-3) \\ &= -2.37 + 3 \\ &= 0.63\end{aligned}$$



$$\begin{array}{r} 3.00 \\ -2.37 \\ \hline 0.63 \end{array}$$

Algebra of Real Functions.

(i) Addition

$$(f+g)(x) = f(x) + g(x)$$

(ii) Subtraction =

$$f(x) - g(x) = (f-g)(x)$$

(iii) multiplication

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

(iv) Division

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad (g(x) \neq 0)$$

(v) Scalar

multiplication

$$(\alpha f)(x) = \alpha \cdot f(x)$$

e.g. $f(x) = 2x + 3 \quad x \in \mathbb{R}$
 $g(x) = x - 7 \quad x \in \mathbb{R}$

① $(f+g)(x) = f(x) + g(x) = 2x + 3 + x - 7 = 3x - 4 \quad , x \in \mathbb{R}$

② $(f-g)(x) = f(x) - g(x) = (2x + 3) - (x - 7) = x + 10 \quad , x \in \mathbb{R}$

③ $(f \cdot g)(x) = (2x + 3)(x - 7) = 2x^2 - 11x - 21 \quad , x \in \mathbb{R}$

④ $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x) \neq 0} = \frac{2x + 3}{x - 7 \neq 0} \quad \checkmark \quad \text{Domain} = x \in \mathbb{R} - \{7\}$

⑤ $(7f)(x) = 7 \cdot f(x) = 7(2x + 3) = 14x + 21 \quad , x \in \mathbb{R}$

$(8g)(x) = 8 \cdot g(x) = 8(x - 7) = 8x - 56 \quad , x \in \mathbb{R}$

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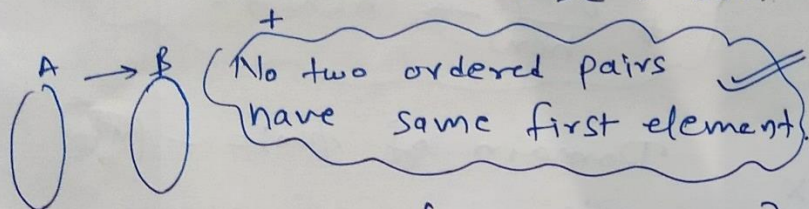
Exercise 2.3 { all questions
are related to
functions }

① (i)

$$\{(2,1), (5,1), (8,1), (11,1), (14,1), (17,1)\}$$

Fn. ✓

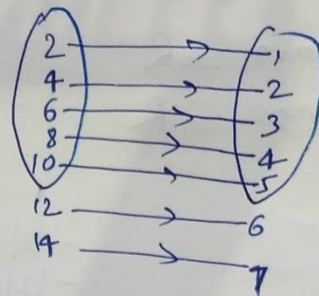
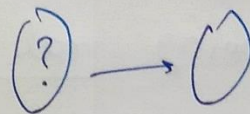
$F_n \rightarrow$ first set should be Domain.



$$\text{Domain} = \{2, 5, 8, 11, 14, 17\}$$
$$\text{Range} = \{1\}$$

(ii)

$$\{(2,1), (4,2), (6,3), (8,4), (10,5), (12,6), (14,7)\}$$



Fn. ✓

$$\text{Domain} = \{2, 4, 8, 6, 10, 12, 14\}$$

$$\text{Range} = \{1, 2, 3, 4, 5, 6, 7\}$$

(iii) $\{(1,3), (1,5), (2,5)\}$

$(1,3), (1,5) \rightarrow$ same first element

$\therefore$ Not a function

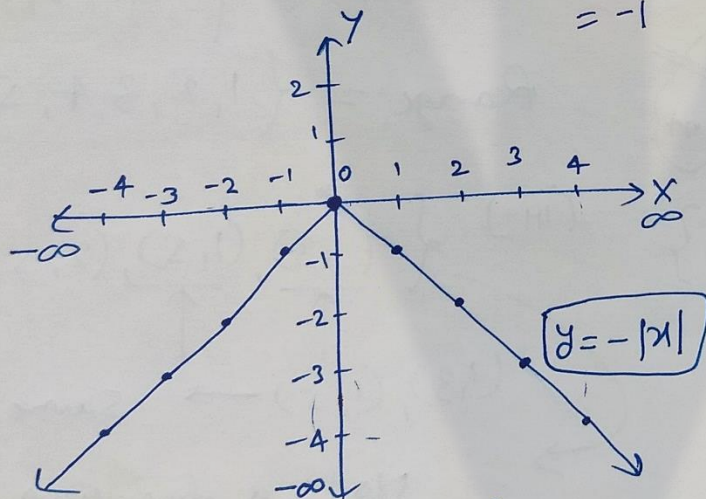
Exercise (2.3)

② Domain and Range of the following real function: $\rightarrow$

input $\rightarrow$ Domain $\subseteq \mathbb{R}$
output $\rightarrow$ Range $\subseteq \mathbb{R}$

(i) $f(x) = -|x| = y$

x	y
0	0 ✓
1	-1 ✓
2	-2 ✓
3	-3 ✓
4	-4 ✓
-1	-1
-2	-2
-3	-3
-4	-4
-5	-5



Domain $= (-\infty, \infty) = \mathbb{R}$

② Range $= (-\infty, 0] \rightarrow -\infty < y \leq 0$

$$\begin{aligned} -|1| &= -1 \\ -|2| &= -2 \\ -|-1| &= -(+1) \\ &= -1 \end{aligned}$$

Second method

$$f(x) = -|x|$$

Domain $\rightarrow$ set of all possible inputs (which can be put in the function)

$\sqrt{g(x)} \rightarrow g(x) \geq 0$

$\frac{g(x)}{h(x)} \rightarrow h(x) \neq 0$

Domain of $f(x) = -|x|$ is $\mathbb{R}$.

Range: $y = f(x) = -|x|$
(Start with known expression)

$$|x| \geq 0 \rightarrow 0 \geq -|x|$$

$$-|x| \leq 0$$

$$\Rightarrow y \leq 0 \Rightarrow -\infty < y \leq 0$$

$$y \in (-\infty, 0]$$

② (ii) $f(x) = \sqrt{9-x^2}$

Domain

(For Real Fun.)

$$9-x^2 \geq 0$$

Coeff. of x^2
 $\downarrow$
 $(+)$

$x=10$

$\rightarrow \sqrt{9-100}$

$\sqrt{-91}$

$\downarrow$
 imaginary

not real

$\Rightarrow$ ~~not~~ multiply $\ominus$

$$x^2 - 9 \leq 0$$

$\Rightarrow$ Factorisation

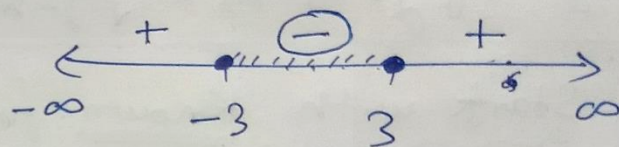
sign of inequality changes

$$\Rightarrow x^2 - 3^2 \leq 0$$

$$\Rightarrow (x-3)(x+3) \leq 0$$

$a^2 - b^2 = (a+b)(a-b)$

Number line
 $\downarrow$
 mark (roots)
 $\swarrow$
 roots
 $x=3$
 $x=-3$



(Sign allotment) $\rightarrow$ right most part $= (+)$

then alternate sign while moving left.

$\checkmark$ Solution $\rightarrow$ part.

(According to sign of inequality)

$$(x-3)(x+3) \leq 0$$

$\ominus$ ve $\leftarrow$ less than or equal to

$\rightarrow$ (corners) include.

$x \in [-3, 3]$
 interval

$-3 \leq x \leq 3$
 Domain

Range of $y = f(x) = \sqrt{9-x^2}$

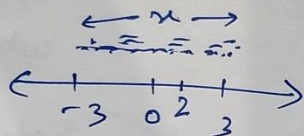
{ Start* with Known function }

By Domain, we Know that

$$x \in [-3, 3]$$

$$\Rightarrow -3 \leq x \leq 3$$

$$\Rightarrow \text{Square}$$



~~$9 \leq x^2 \leq 9$~~ ~~$x^2 = 9$~~

$$x=2$$

$$x^2=4$$

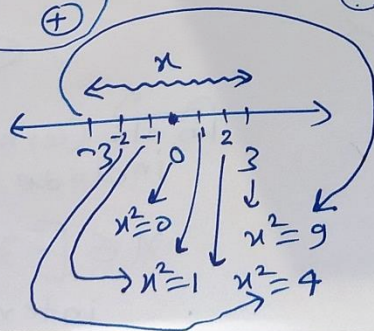
$$x^2 = x \cdot x = \begin{matrix} + & + \\ - & - \end{matrix} = \begin{matrix} + \\ + \end{matrix}$$

$$-3 \leq x \leq 3$$

$$\downarrow$$

$$0$$

$$0 \leq x^2 \leq 9$$



Universal truth
for real x

$$x^2 \geq 0$$

$$0 \leq x^2 \leq 9$$

multiply with $\ominus$

$$\Rightarrow 0 \geq -x^2 \geq -9$$

$$\Rightarrow -9 \leq -x^2 \leq 0$$

$$\Rightarrow 9-9 \leq 9-x^2 \leq 9+0$$

$$\Rightarrow 0 \leq 9-x^2 \leq 9$$

Square root $\pm \sqrt{x}$

$$0 \leq \sqrt{9-x^2} \leq 3$$

$$\Rightarrow 0 \leq y \leq 3$$

$$\text{Range} = [0, 3]$$

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Exercise 2.3 x Relations & Functions

③ $f(x) = 2x - 3$

(i) $f(0) = 2(0) - 3 = 0 - 3 = -3$

(ii) $f(7) = 2(7) - 3 = 14 - 3 = 11$

(iii) $f(-3) = 2(-3) - 3 = -6 - 3 = -9$

④

input $\downarrow$ $t(c) = \frac{9c}{5} + 32$ $\uparrow$ output $\parallel$ F

't' is a function to change Celsius into Fahrenheit.

(i) $t(0) = \frac{9(0)}{5} + 32 = 32$

$\uparrow$ 0°Celsius Fahrenheit

(ii) $t(28)$

$$t(c) = \frac{9c}{5} + 32$$

$$t(28) = \frac{9 \times 28}{5} + 32$$

$$= \frac{252}{5} + \frac{32}{1} = \frac{252 + 160}{5}$$

$$= \frac{412}{5} \text{ Fah.}$$

(iii) $t(-10) = \frac{9(-10)}{5} + 32$

$$= 9(-2) + 32 = -18 + 32 = 14$$

(iv) The value of c , when $t(c) = 212$

$$\frac{9c}{5} + 32 = 212$$

$$\Rightarrow \frac{9c}{5} = 180$$

$$\Rightarrow c = 20 \times 5 = 100$$

⑤ Find the range:

(i) $f(x) = 2 - 3x$, $x \in \mathbb{R}$, $x > 0$

(Start with Known Function)

$y = f(x) = 2 - 3x$

We know that $x > 0$ (Given)

⇒ multiply $(-3) = \text{Ove}$

⇒ $-3x < 0$

add 2

⇒ $2 - 3x < 2 + 0$

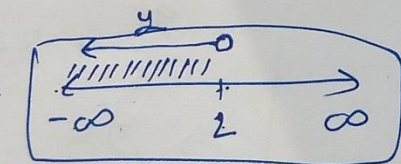
sign
change

Inequality

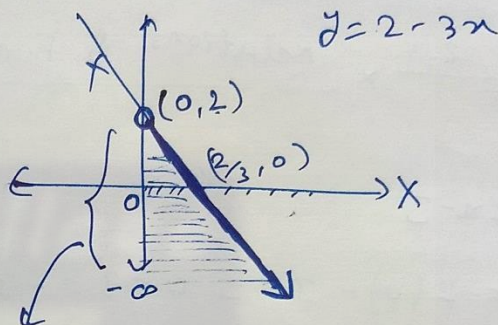
⇒ $2 - 3x < 2$

$f(x) < 2$

$-\infty < y < 2$



$y \in (-\infty, 2)$



Range
 $(-\infty, 2)$

Right side graph

(ii) $f(x) = x^2 + 2$,

$x \in \mathbb{R}$

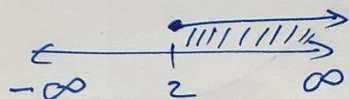
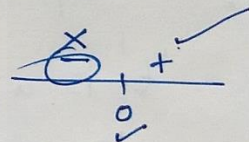
x is any
real
no.

We know that

$x^2 \geq 0$

⇒ $x^2 + 2 \geq 0 + 2$

$y = f(x) \geq 2$



Range $= [2, \infty)$

(iii) $y = f(x) = x$, $x \in \mathbb{R}$ (Identity F_n)

⇒ $x \in \mathbb{R}$

⇒ $y \in \mathbb{R}$

$y = \text{out put} = \text{Range}$

⇒ Range $= \mathbb{R}$

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Miscellaneous Exercise (2)

Chapter - 2

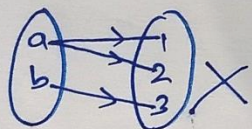
Relations and Functions

①

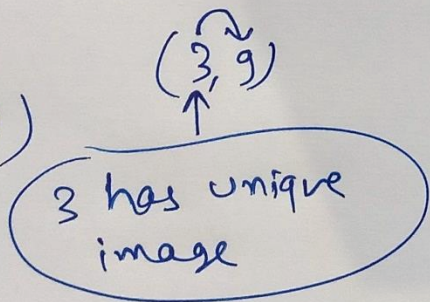
$$f(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 3x, & 3 \leq x \leq 10 \end{cases}$$

$$g(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 3x, & 2 \leq x \leq 10 \end{cases}$$

$$f(3) = 3^2, (x^2) \\ = 9$$



$$f(3) = 3 \times 3, (3x) \\ = 9$$



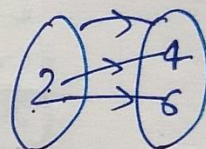
$$g(2) = 2^2 \\ = 4$$

(according to x^2)

$$g(2) = 3(2) \\ = 6$$

(according to $3x$)

$$(2, 4), (2, 6)$$



2 doesn't have unique image.

$\therefore f$ is a function
but g is only a relation.

② $f(x) = x^2$

$$\frac{f(1.1) - f(1)}{1.1 - 1} = \frac{(1.1)^2 - (1)^2}{1.1 - 1} = \frac{1.21 - 1}{0.1} \\ = \frac{0.21}{0.1} = 2.1$$

③ Find the domain of the function

$$f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12} = \frac{Nx}{Dx \neq 0}$$

Domain:

$$\begin{array}{l} \downarrow \sqrt{f(x)} \rightarrow \underline{f(x) \geq 0} \\ \downarrow \frac{f(x)}{g(x)} \rightarrow \underline{g(x) \neq 0} \end{array}$$

For Domain

$$\underline{\text{Denom.}} = x^2 - 8x + 12 \neq 0$$

$$\Rightarrow x^2 - 2x - 6x + 12 \neq 0$$

$$\Rightarrow x(x-2) - 6(x-2) \neq 0$$

$$\Rightarrow (x-2)(x-6) \neq 0$$

$$x \neq 2, x \neq 6$$

x can be anything but
not 2 & 6.

$$x \in \mathbb{R} - \{2, 6\} = \underline{\text{Domain}}$$

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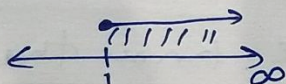
Miscellaneous Exercise Chapter (2)

④ $f(x) = \sqrt{x-1}$

Domain Square root

$$x-1 \geq 0$$

$$\Rightarrow x \geq 1$$



Domain = $[1, \infty)$ $x \in [1, \infty)$

Range: $y = \sqrt{x-1}$

x	1	2	3	4	5	10
y	0	1	$\sqrt{2}$	$\sqrt{3}$	2	3

$y \in [0, \infty)$

Range: start with known function.

Due to Domain

$$x-1 \geq 0$$

$$\sqrt{x-1} \geq 0 \Rightarrow y \geq 0$$

⑤ $f(x) = |x-1|$

Domain

neither $\sqrt{\dots}$
nor $\frac{N}{D}$

$$x \neq \ominus$$

$$x \neq \oplus$$

$$x \neq 0$$

we can put all real
numbers.

Domain = $x \in \mathbb{R}$

Range. $y = |x-1|$

$|x| \geq 0$

we know that

$$|x-1| \geq 0$$

$$y \geq 0$$

$$\Rightarrow y \in [0, \infty)$$

x	y
0	1
1	0 ✓
2	1
3	2
-1	2
-2	3
-3	4

0, $\oplus$ we

Range = $[0, \infty)$

⑥ $f = \left\{ \left(x, \frac{x^2}{1+x^2} \right) : x \in \mathbb{R} \right\}$

$f = \left\{ \left(\underset{\substack{\uparrow \\ \text{ordered pair}}}{x, y} \right) : x \in \mathbb{R}, \text{ relation b/w } x \text{ \& } y \right\}$

$\uparrow$
 f_n^m

$f = \left\{ \left(\underline{x, y} \right) : \underline{x \in \mathbb{R}}, \underline{y = \frac{x^2}{1+x^2}} \right\}$

$f(x) = y = \frac{x^2}{1+x^2}, \quad x \in \mathbb{R}$

$\uparrow$

I - method Range

Start with known f_n^m .

$$y = \frac{x^2}{1+x^2}$$

$$y = \frac{\overbrace{x^2+1}^{1+x^2} - 1}{1+x^2} = \frac{x^2+1}{x^2+1} - \frac{1}{1+x^2}$$

$$y = 1 - \frac{1}{1+x^2}$$

Single place of x

Start with known f_n^m .

we know that $x^2 \geq 0$

$$\Rightarrow 1+x^2 \geq 1+0$$

$$\Rightarrow \infty > 1+x^2 \geq 1$$

Reciprocal. (Sign of In. $\rightarrow$ change)

$$\frac{1}{\infty} = 0$$

$$\frac{1}{1} = 1$$

$$\Rightarrow 0 < \frac{1}{1+x^2} \leq 1$$

$$\Rightarrow 0 > -\frac{1}{1+x^2} \geq -1$$

$$\Rightarrow 1 > \boxed{1 - \frac{1}{1+x^2}} \geq 1-1$$

$$\Rightarrow 1 > y \geq 0$$

$$\Rightarrow 0 \leq y < 1$$

$$y \in [0, 1)$$

II - method. Range=? $x \in \mathbb{R}$

Represent 'x' in terms of 'y'

$$y = \frac{x^2}{1+x^2}$$

Now
y is represented
in terms of x

$$\Rightarrow y + y \cdot x^2 = x^2$$

$$\Rightarrow y = \frac{-y \cdot x^2 + x^2}{1}$$

$$\Rightarrow x^2 \cdot (-y + 1) = y$$

$$\Rightarrow x^2 = \frac{y}{1-y}$$

$$\Rightarrow x = \pm \sqrt{\frac{y}{1-y}}$$

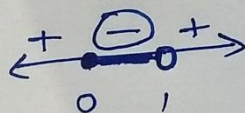
Given
 $x \in \mathbb{R}$

x is Real.

$\sqrt{-}$ x

$$\sqrt{\frac{y}{1-y}} \rightarrow \text{Real}$$

$$\frac{y}{1-y} \geq 0$$



$$y \in [0, 1)$$

$$\frac{y}{1-y} \geq 0$$

$$\Rightarrow \frac{y}{1-y} \leq 0$$

$$y=0 \rightarrow y=0$$

$$y-1=0 \rightarrow y=1$$

Always Exclude

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Misc. Chapter (2)

$$\textcircled{7} \quad f, g: \mathbb{R} \rightarrow \mathbb{R} \quad \left| \begin{array}{l} f(x) = x+1 \\ g(x) = 2x-3 \end{array} \right.$$

$f+g$

↓

$$(f+g)(x) = f(x) + g(x)$$

$$= (x+1) + (2x-3)$$

$$= 3x-2 \quad \checkmark \quad (x \in \mathbb{R})$$

$$f-g \rightarrow (f-g)(x) = f(x) - g(x)$$

$$= (x+1) - (2x-3)$$

$$= -x+4 \quad \checkmark \quad (x \in \mathbb{R})$$

$$\frac{f}{g} \rightarrow \frac{f(x)}{g(x)} = \frac{x+1}{2x-3} \quad \checkmark$$

$$D_r \neq 0$$

$$2x-3 \neq 0$$

$$x \neq \frac{3}{2}$$

$$x \in \mathbb{R} - \left\{ \frac{3}{2} \right\}$$

$$\textcircled{8} \quad f = \{(1,1), (2,3), (0,-1), (-1,-3)\}$$

$$\boxed{z \rightarrow z}$$

$$\rightarrow (x,y)$$

$$y = f(x) = ax+b$$

$$(1,1)$$

$$\text{Domain} = \{1, 2, 0, -1\}$$

$$\text{Range} = \{1, 3, -1, -3\}$$

$$1 = ax+1+b$$

$$\Rightarrow \boxed{a+b=1} \quad \textcircled{1}$$

$$y = ax+b$$

$$(0,-1) \equiv (x,y)$$

$$\Rightarrow -1 = a(0) + b$$

$$\Rightarrow -1 = 0 + b$$

$$\Rightarrow \boxed{b=-1} \quad \textcircled{2}$$

$$b = -1 \quad \checkmark$$

$$\text{By eq. } \textcircled{1}$$

$$a + (-1) = 1$$

$$\Rightarrow \boxed{a=2} \quad \checkmark$$

⑨ R relation from N to N

$$R = \{(a, b), a, b \in N, a = b^2\} \rightarrow (a, b) \in R$$

(i) $(a, a) \in R$, for all $a \in N$

$$\begin{cases} (1, 1) \in R \\ 1 = 1^2 \checkmark \end{cases} \quad a = 1, 2, 3, 4, \dots$$

$$\begin{cases} (2, 2) \in R \\ 2 = 2^2 \times \end{cases} \quad \text{False}$$

when $(a, b) \in R \Rightarrow a = b^2$

$$(a, a) \in R \Rightarrow a = a^2$$

$$\Rightarrow a^2 - a = 0$$

$$\Rightarrow a(a-1) = 0$$

$$\downarrow \quad \downarrow$$

$$a = 0$$

$$\times$$

$$0 \notin N$$

$$a = 1 \in N$$

$$\uparrow$$

only one value of a ,

not for all $a \in N$

False

(ii) $(a, b) \in R$, implies $(b, a) \in R$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ a = b^2 & \not\rightarrow & b = a^2 \end{array}$$

$$\text{Let } a = 9 \mid b = 3 \mid \underline{9 = 3^2}$$

$$(a, b) = (9, 3) \in R$$

$$(b, a)$$

$$(3, 9) \in R ???$$

False

$$3 = 9^2$$

$$\Rightarrow 3 = 81 \times$$

(iii) if $(a, b) \in R, (b, c) \in R$ implies $(a, c) \in R$

$$\begin{array}{ccc} \leftarrow \downarrow \text{let} & \downarrow & \downarrow \text{Prove} \\ a = (b^2) & b = c^2 & \boxed{a = c^2} \\ \text{???} & & \end{array}$$

$$a = (c^2)^2$$

$$\Rightarrow \boxed{a = c^4}$$

False

(10)

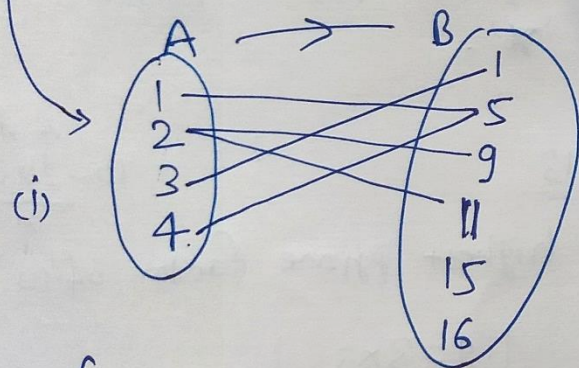
$$A = \{1, 2, 3, 4\}$$

$$B = \{1, 5, 9, 11, 15, 16\}$$

$$f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$$

(i) f is a relation from A to B

(ii) f is a function from A to B .



True.

f is a relation from A to B .

$$\text{Domain} = \{1, 2, 3, 4\} = A$$

$$\text{Range} = \{1, 5, 9, 11\} \subset B$$

(ii)

function

(i) 1st set $A = \text{Domain}$ ✓

(ii) each element of set A

↓
must have unique image in set B .

2 has 2 images.

$\Rightarrow f$ is not a function

~~(11) f is subset of $\mathbb{Z} \times \mathbb{Z}$~~
 ~~$f = \{ \}$~~

⑪ f is subset of $\mathbb{Z} \times \mathbb{Z}$

~~$f = \{(a, b), (a, b)\}$~~

$f = \{(\overbrace{ab}, \overbrace{a+b}) : a, b \in \mathbb{Z}\}$

$f = \{(x, y) : \text{---}\}$

Is f function from $\mathbb{Z}$ to $\mathbb{Z}$?

$(x, y) \in f$

$10 = x = ab$

$y = a + b$

$a, b \in \mathbb{Z}$

Let $a = 10, b = 1$

$a \times b = 10 \times 1 = 10 = x$

$y = a + b = 10 + 1 = 11$

$(x, y) \in f$

$(10, 11) \in f$

$(10, 7) \in f$

Let $a = 2, b = 5$

$x = ab = 2 \times 5 = 10$

$y = a + b = 2 + 5 = 7$

$\therefore$ '10' has 2 images.

$\Rightarrow f$ is not a function.

⑫ $A = \{9, 10, 11, 12, 13\}$

$f: A \rightarrow N$

$f(n)$ = the highest prime factor of n .

$f(n)$ = the highest prime factor of 'n'.

input. A

$x = 9, 10, 11, 12, 13$

$f(10)$ = the highest prime factor of 10 = 5

$f(9) = 3$

$f(11) = 11$

$f(12) = 3$

$f(13) = 13$

$9 = 3 \times 3$

$11 = 1 \times 11$

$12 = 2 \times 3 \times 2$

$13 = 1 \times 13$

Range

$= \{3, 5, 11, 13\}$



$10 = 2 \times 5$

All Useful Links:

- **YouTube Channel : Toppers Village** (Get full and free video lectures of Class 10 & 11 Maths) - <https://www.youtube.com/toppersvillage>
- **Website :** www.toppersvillage.com
- **Instagram :** <https://www.instagram.com/toppersvillage/>
- **Facebook Page:** www.facebook.com/ToppersVillage/
- **Facebook Group:** <https://www.facebook.com/groups/ToppersVillage>